B. Sc. B.Ed. SEMESTER I EXAMINATION 2019

Subject: Physics

GE1/ GE2 (Mathematical Physics - I)

FULL MARKS: 50 TIME ALLOWED: 2 HOURS

Answer any **Ten** (10) questions

1. (i) Find $\frac{dy}{dx}$ when $x^3 + y^3 = 3axy$ where a is a constant. (2 marks)

(ii) Determine whether the following equation is exact
$(4x^3 + 6xy + y^2)\frac{dy}{dx} = -(3x^2 + 2xy + 2)$ (3 marks)

2. (i) Prove that if $y^3 - 3ax^2 + x^3 = 0$ then $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$ (2 marks)

(ii) Find $\frac{dz}{dt}$ using chain rule if $z = xy^2 + x^2y$, $x = at^2$, $y = 2at$ (3 marks)

3. State and prove Gauss' Divergence theorem. (5 marks)

4. (i) Find the projection of vector $\vec{A} = \hat{i} - 2\hat{j} + \hat{k}$ on the vector $\vec{B} = 4\hat{i} - 4\hat{j} + 7\hat{k}$. (2 marks)

(ii) Show that $\vec{a} \cdot (\vec{b} \times \vec{c})$ in its absolute value equal to the volume of the parallelepiped of sides a, b and c. (3 marks)

5. For three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ prove that $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$ (5 marks)

6. If $\Phi(x, y, z) = xy^2z$ and $\vec{A} = xz\hat{i} - xy^2\hat{j} + yz^2\hat{k}$ then find $\frac{\partial^3}{\partial x^2 \partial z}(\Phi\vec{A})$ at $(2, 0, 1)$

7. (i) Define Gradient. (1 marks)

(ii) What does it mean physically? (1 marks)

(iii) If $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$ then find $\boldsymbol{\nabla}\left(\frac{1}{R}\right)$. (3 marks)

8. Prove that

(i) $\boldsymbol{\nabla} \times \boldsymbol{\nabla}\phi = 0$ (2 marks)

(ii) $\boldsymbol{\nabla} \cdot \boldsymbol{\nabla} \times \vec{A} = 0$ (3 marks)

9. (i) Define a Vector Space. (2 marks)

(ii) What do you mean by the basis of a Vector Space. (1 marks)

(iii Define orthonormal and orthogonal set of vectors. (2 marks)

10. (i) Define eigenvalues and eigenvectors. (2 marks)

(ii) Find the eigenvalues of $X = \begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}$ (3 marks)

11. (i) Justify with reason whether the following equation is homogeneous $\frac{dy}{dx} = \frac{2x-y}{x-3y}$ (2 marks)

(ii) Solve the following:
$(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0,\ y(0) = 1$ (3 marks)

12. Solve the following:
$m\frac{d^2x}{dy^2} + \alpha\frac{dy}{dx} + \beta x = k\cos(\omega x),\ y(0) = 0$ (5 marks)